critical heat flux, W/m^2; τ_{imp}, duration of thermal pulse, sec; τ_{cr}, interval of time from moment of increase in thermal load to onset of sheet boiling, sec.

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FRICTION AND HEAT TRANSFER IN THE TURBULENT FLOW OF A COMPRESSIBLE GAS IN A PLANE CHANNEL

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Results are given of calculating the local friction and heat-transfer characteristics for the case of gas cooling. The calculations are compared with experiment.

At present, the problem of calculating the resistance and heat transfer in the flow of liquid and compressible gas with variable thermophysical properties through channels is attracting great interest; see [1-5], etc. The so-called stabilized profile is often used as initial velocity distribution here. For compressible-gas flow, this method, like the division of the flow into initial (stabilization section) and basic (stabilized section) sections over the length of the channel, is arbitrary.

In the present work, the results of numerical integration of the system of equations in the boundary-layer approximation, describing steady turbulent flow of compressible gas in the inlet section of a plane-parallel channel, are given. The hypothesis of hydrodynamic stabilization is not used here.

Introducing the dimensionless quantities

$$\bar{x} = \frac{x}{h\,\mathrm{Re}_\mathrm{I}},\quad \bar{y} = \frac{y}{h},\quad \bar{u} = \frac{u}{u_\mathrm{I}},\quad \bar{v} = \frac{v}{u_\mathrm{I}}\mathrm{Re}_\mathrm{I},\quad \Theta = \frac{T_0}{T_{0\mathrm{I}}},$$

$$\bar{p} = \frac{p_\mathrm{I} - p}{\rho_\mathrm{I} u_\mathrm{I}^2},\quad \bar{\rho} = \frac{\rho}{\rho_\mathrm{I}},\quad \bar{\mu} = \frac{\mu}{\mu_\mathrm{I}},\quad \xi_\mathrm{I} = \frac{u_\mathrm{I}}{\sqrt{2c_p T_{0\mathrm{I}}}},$$

$$\mathrm{Re}_\mathrm{I} = \frac{\rho_\mathrm{I} u_\mathrm{I} h}{\mu_\mathrm{I}},$$

the equations of motion, energy, continuity, and constancy of flow rate take the form

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu_e \frac{\partial u}{\partial y}\right), \tag{1}$$

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$$\rho\left(u\frac{\partial\Theta}{\partial x}+v\frac{\partial\Theta}{\partial y}\right)=\frac{\partial}{\partial y}\left(\frac{\mu_e}{\mathrm{Pr}_e}\frac{\partial\Theta}{\partial y}\right)+\frac{\partial}{\partial y}\left[\left(1-\frac{1}{\mathrm{Pr}_e}\right)\mu_e\xi_I^2\frac{\partial u^2}{\partial y}\right], \quad (2)$$

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0, \quad (3)$$

$$\int_0^1 \rho u dy=1. \quad (4)$$

The system in Eqs. (1)-(4) describes the turbulent flow under the assumption that

$$\tau=\mu_e\frac{\partial u}{\partial y},\quad q=-\lambda_e\frac{\partial T}{\partial y},$$

where

$$\mu_e=\mu+\mu_t;\quad \lambda_e=\lambda+\lambda_t.$$

In the case of a perfect gas, the equation of state is added to the system in Eqs. (1)-(4)

$$\rho=\frac{1-\xi_I^2}{\Theta-\xi_I^2u^2}\left(1-\frac{2k}{k-1}\frac{\xi_I^2}{1-\xi_I^2}p\right). \quad (5)$$

The thermophysical properties of the gas are found from the relations

$$\mu=\frac{\Theta-\xi_I^2u^2}{1-\xi_I^2},\quad c_p=\mathrm{const},\quad \mathrm{Pr}=1. \quad (6)$$

The turbulent viscosity μ_t is found from the Prandtl hypothesis with damping by the Van Driest comfort factor [6]

$$\mu_t=0.16\rho y^2\left[1-\exp\left(\frac{-y\sqrt{\tau\rho\,\mathrm{Re}_I}}{26\mu}\right)\right]\left|\frac{\partial u}{\partial y}\right|\mathrm{Re}_I \text{ when } 0<y\leqslant 0.25y_\delta, \quad (7)$$

$$\mu_t=0.01\rho y_\delta^2\left|\frac{\partial u}{\partial y}\right|\mathrm{Re}_I \text{ when } 0.25y_\delta<y\leqslant 1,$$

where y_δ is the distance from the wall at which the velocity differs from that at the flow core by no more than 1%. The turbulent Prandtl number is taken to be unity.

The boundary conditions are specified as follows

$$u(0,y)=\Theta(0,y)=1,$$

$$u(x,0)=v(x,0)=0,\quad \frac{\partial u(x,1)}{\partial y}=0, \quad (8)$$

$$\Theta(x,0)=\mathrm{const},\quad \frac{\partial\Theta(x,1)}{\partial y}=0.$$

The system in Eqs. (1)-(5), together with the boundary conditions in Eq. (8) and the dependences for the transfer coefficients in Eqs. (6) and (7), is solved numerically by a difference method. A two-layer six-point implicit difference scheme on a nonuniform grid is used, in combination with the fitting method [7]. The step Δx along the x axis is chosen by testing and remains constant; the step Δy is variable. The step Δy is chosen so that it is as small as possible close to the wall and increases on moving away. The first few steps Δy_i are constant, while the next form a geometric progression with the common ratio $n = \Delta y_{i+1}/\Delta y_i$. The number of steps along the y axis is chosen so that the value of n is close to unity for local retention of accuracy [8], while several steps Δy_i fall simultaneously within any sublayer.

The velocity, temperature, pressure, pressure gradient, frictional stress, and heat-flux density are determined in the calculations. The friction and heat-transfer coefficients are found from the relations

$$c_f\mathrm{Re}_I=\frac{2\tau_w}{u_m},\quad \mathrm{St}\,\mathrm{Re}_I=\frac{q_w}{T_m-\Theta_w},$$

where

TABLE 1. Comparison of Calculated and Experimental Values of Frictional Stress at the Wall

Case No.	1	2	3	4
ξ_I	0,266	0,268	0,288	0,322
Θ_w	0,2	0,234	0,238	0,313
Re_I	$8\cdot10^2$	$1,27\cdot10^3$	$2,36\cdot10^3$	$3,65\cdot10^3$
$\bar{\tau}_w$exp, N/m²	17	21,5	31	35
$\bar{\tau}_{wc}$, N/m²	17,3	20,5	33,8	38

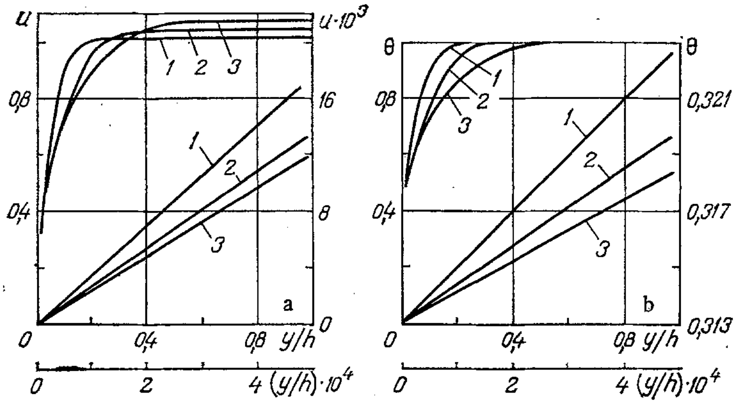

Fig. 1. Velocity (a) and stagnation-temperature (b) profiles in three channel cross sections: 1-3) x/h = 2, 12, 22.

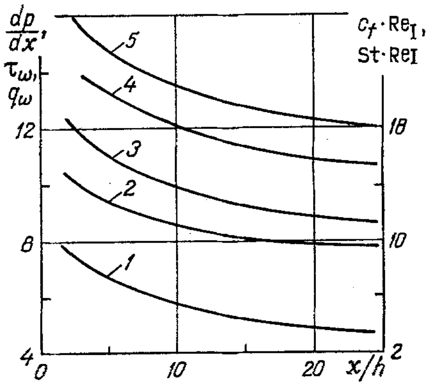

Fig. 2

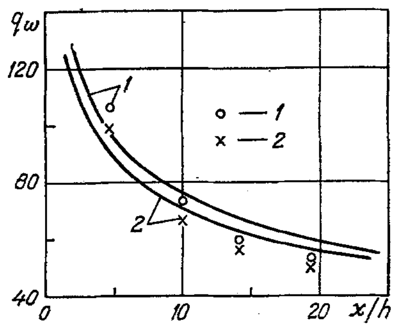

Fig. 3

Fig. 2. Variation in flow and heat-transfer characteristics along the channel: 1) q_w; 2) St·Re_I; 3) τ_w; 4) dP/dx; 5) $c_f Re_I$.

Fig. 3. Variation in heat-flux density along channel: 1) case No. 1 from Table 1 according to calculation (curve) and experiment (points); 2) case No. 2 from Table 1 according to calculation (curve) and experiment (points). q_w, kW/m².

$$u_m = \int_0^1 \rho u^2 dy, \quad T_m = \int_0^1 \rho u(\Theta - \xi_I^2 u^2)\, dy$$

are the mean-mass velocity and temperature, respectively.

Below, data are given for calculations using the same values of the dimensionless inlet velocity ξ_I, Reynolds number Re_I, and temperature ratio T_w/T_{0I} as in the experiments of [9]

(Table 1). Numerical results for case 4 are shown in Figs. 1 and 2. The character of the deformation in the velocity and stagnation-temperature profiles (Fig. 1) indicates a slow growth in the dynamic and thermal boundary layers, accompanied by a simultaneous decrease in the flow core and an increase in the velocity there with increasing distance from the channel inlet. In the calculations, as in the experiments of [9] in measuring the velocity profiles close to the channel outlet in the case of a relative length x/h = 22, the opposing boundary layers did not meet. Even in the cross section x/h = 30, at which the calculations ended, the flow core remains a considerable part of the channel cross section. The chosen step size Δy_i along the y axis offers the possibility of finding the velocity and temperature distribution in any sublayer. The distributions in Fig. 1 correspond to the lower abscissa and the right-hand ordinate. Analysis of the results shows that in the case of gas cooling in the given flow section no marked influence of heat transfer on the velocity profile is observed.

The smooth variation in the local values of the stress τ_w and heat-flux density q_w and also of the frictional coefficient c_f and the heat-transfer coefficient St (Fig. 2) is due to the weak decrease in pressure gradient dp/dx over the length of the channel. This also explains why the disruption of the Reynolds analogy at such values of dp/dx is slight. The ratio $c_f/\text{St} \cong 1.85$, which is close to 2.

Table 1 gives the results of comparing the frictional stress at the wall obtained in the calculations with experimental measurements using a mobile element [9]. In the experiments, the mean value over the length of the mobile element $\bar{\tau}_{w\text{exp}}$ was determined, while the mean in the calculations $\bar{\tau}_{wc}$ was found using the local values τ_w from the formula

$$\bar{\tau}_{wc} = \frac{1}{l} \int_0^l \tau_w(x)\, dx.$$

Satisfactory agreement of the heat-flux densities obtained in the calculations and in the experiment of [9] is seen in Fig. 3 for two cases of flow. The values of Θ_w and Re_I in those cases are shown in Table 1.

Comparison of the results of calculation and experiment confirms the reliability of the method employed. The distributions of the local flow and heat-transfer characteristics given here, together with the experimental data of [9], give a sufficiently complete picture of the resistance and heat transfer in the given channel.

NOTATION

c_p, specific heat at constant pressure; c_v, specific heat at constant volume; c_f, frictional coefficient; 2h, channel height; $k = c_p/c_v$, adiabatic modulus; p, pressure, Pr, Prandtl number; q, specific heat flux; Re, Reynolds number, St, Stanton number; T, temperature; u, v, longitudinal and transverse velocity components; x, y, longitudinal and transverse coordinates; λ, thermal conductivity; μ, dynamic viscosity; ρ, density; τ, tangential stress. Indices: 0, stagnation parameters; I, inlet; m, mean-mass quantities, t, turbulent; c, calculation; w, channel wall; e, effective; exp, experiment.

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HYDRODYNAMICS AND HEAT EXCHANGE IN VISCOUS INCOMPRESSIBLE LIQUID FLOW BETWEEN DISKS ROTATING IN A CYLINDRICAL CASING

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The article provides the results of a numerical investigation of convective heat exchange in a closed system consisting of two coaxial disks rotating at the same angular velocity within an immobile cylindrical casing.

Improvement in the reliability of calculations of the thermal and thermal-stress conditions of turbomachine rotors requires, in particular, greater accuracy in assigning the boundary conditions of heat exchange at their end-face surfaces. Most of the theoretical, including numerical, investigations in this field were based on mathematical models utilizing cut-off parabolic Navier—Stokes equations. However, in many actual channels adjacent to the rotor surface, inertial forces produce return flow, which cannot be calculated by using methods based on boundary layer theory. A simplified model of one of such systems would be a cavity formed by an immobile cylindrical casing and two disks rotating at a constant angular velocity. This problem has been solved in [1-3] for small Reynolds numbers ($\mathrm{Re} < 2\cdot 10^3$) in the absence of heat exchange.

In stating our problem here, we use the same simplifying assumptions as in [2, 3]: Steady-state laminar flow is contemplated, the velocity and temperature fields are assumed to be axisymmetric, and the thermophysical characteristics of the medium are considered to be constant.

The flow geometry and the coordinate system are shown in Fig. 1.

The system of differential equations of convective heat exchange, written in terms of dimensionless variables, can be conveniently reduced to four equations with identical structures [4]:

$$a\left\{\frac{\partial}{R\partial R}\left(b\,\frac{\partial}{\partial R}\,(c\Phi)\right)+\frac{\partial^2\Phi}{\partial Z^2}\right\}-\frac{1}{R}\left\{\frac{\partial}{\partial R}(RV_r\Phi)+\frac{\partial}{\partial Z}\,(RV_z\Phi)\right\}+d=0, \tag{1}$$

where ω/R, RV_φ, Ψ, and T are considered as the sought function Φ. The corresponding values of the coefficients a, b, c, and d determining the actual form of the equations in the system are given in Table 1.

The r_0 and $2\pi n r_0$ values are used as the coordinate and velocity scales, respectively.

The solution of the system of equations (1) must satisfy the following boundary conditions:

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